This paper aims to achieve two things: to identify the conditions for transitivity in probabilistic support in various settings, and to uncover the components and structure of the mediated probabilistic relation. It is shown that when the probabilistic relation between the two propositions $x$ and $z$ is mediated by multiple layers of partitions of propositions, the impact $x$ has on $z$ consists of the purely indirect impact, the purely bypass impact, and the mixed impact. It is also shown that although mediated confirmation as a whole is not transitive, the indirect part of mediated confirmation is transitive.

1. Introduction
This paper aims to achieve two things. The narrower objective is to identify the conditions for transitivity in probabilistic support in various settings. The broader objective is to uncover the components and structure of the mediated probabilistic relation.

To begin with the former, it is well known that probabilistic support is non-transitive in the following sense. Let $x$, $y$, $z$ be propositions that are not certain, that is, $0 < \Pr(x)$, $\Pr(y)$, $\Pr(z) < 1$.\footnote{This is assumed throughout the paper.} Suppose $x$ probabilistically supports $y$ and $y$ in turn probabilistically supports $z$, where ‘probabilistic support’ means raising the probability, or incremental confirmation (hereafter simply ‘confirmation’). It does not follow from these suppositions that $x$ confirms $z$, that is, there is no guarantee that $x$ raises the probability of $z$. To express it formally, $\Pr(y) < \Pr(y \mid x)$ and $\Pr(z) < \Pr(z \mid y)$ together do not entail that $\Pr(z) < \Pr(z \mid x)$. To illustrate the point, suppose a fair die is rolled. The truth of the proposition $x$ that the die shows a non-large number (not a six or a five)
raises the probability of \( y \) that the die shows a non-extreme number (not a six or a one) from 2/3 to 3/4, and \( y \) in turn raises the probability of \( z \) that the die shows a non-small number (not a one or a two) from 2/3 to 3/4. However, the truth of the proposition \( x \) that the die shows a non-large number reduces the probability of \( z \) that the die shows a non-small number from 2/3 to 1/2.

Non-transitivity of confirmation is a source of concern since we are accustomed to transitivity in logical entailment and tend to assume the same even when the support is only probabilistic. Fortunately, there are conditions under which confirmation is known to be transitive. For example, confirmation is transitive when the mediating pair of propositions \( Y = \{ y, \neg y \} \) screens off the supporting proposition \( (x) \) from the supported proposition \( (z) \) as follows (Shogenji [2003]).

\[
TC^e \quad \text{For any propositions } x, y, z \text{ if } Pr(y) < Pr(y \mid x) \text{ and } Pr(z) < Pr(z \mid y), \text{ then } Pr(z) < Pr(z \mid x), \text{ provided } Y = \{ y, \neg y \} \text{ screens off } x \text{ from } z, \text{ that is, provided } Pr(z \mid y \land x) = Pr(z \mid y) \text{ and } Pr(z \mid \neg y \land x) = Pr(z \mid \neg y).
\]

To express the sense of ‘\( Y = \{ y, \neg y \} \) screens off \( x \) from \( z \)’ informally, once the truth value of the mediating proposition \( y \) is given, the truth of \( x \) becomes irrelevant to the probability of \( z \). This means that the truth of \( x \) affects the probability of \( z \) only indirectly through its impact on the probability distribution over \( Y = \{ y, \neg y \} \). So, one way of understanding \( TC^e \) is that confirmation is transitive provided the confirmation of \( z \) by \( x \) is entirely indirect through \( Y = \{ y, \neg y \} \).

The principle \( TC^e \) is a consequence of (1) below, which holds under the screening-off condition (Shogenji [2003]).

\[
Pr(z \mid x) - Pr(z) = \frac{[Pr(y \mid x) - Pr(y)][Pr(z \mid y) - Pr(z)]}{1 - Pr(y)} \quad (1)
\]

It is easy to see from (1) why confirmation is transitive under the screening-off condition. When \( x \) confirms \( y \) and \( y \) confirms \( z \), the numerator \([Pr(y \mid x) - Pr(y)][Pr(z \mid y) - Pr(z)]\) is positive. Since the denominator \( 1 - Pr(y) \) is also positive from the assumption that \( 0 < Pr(y) < 1 \), we can conclude that \( Pr(z \mid x) - Pr(z) \) is positive, and thus \( x \) confirms \( z \).

Roche ([2012]) extended Shogenji’s result to show that transitivity holds under a weaker condition (the negative impact screening-off condition) in \( TC^s \) below by way of proving (2) under the weaker condition.

\[
TC^s \quad \text{For any propositions } x, y, z \text{ if } Pr(y) < Pr(y \mid x) \text{ and } Pr(z) < Pr(z \mid y), \text{ then } Pr(z) < Pr(z \mid x), \text{ provided } Y = \{ y, \neg y \} \text{ screens off the negative impact of } x \text{ from } z, \text{ that is, } Pr(z \mid y \land x) \geq Pr(z \mid y) \text{ and } Pr(z \mid \neg y \land x) \geq Pr(z \mid \neg y).
\]

\[
Pr(z \mid x) - Pr(z) \geq \frac{[Pr(y \mid x) - Pr(y)][Pr(z \mid y) - Pr(z)]}{1 - Pr(y)} \quad (2)
\]

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[2] A terminological side note. We use ‘an impact’ as a generic term while reserving ‘confirmation’ for cases of a positive impact where the conditional probability is greater than the marginal probability. There is therefore no ‘negative confirmation’.
The negative impact screening-off condition allows the truth of \( x \) to affect the probability of \( z \) even when the truth value of \( y \) is given, but the impact cannot be negative. The standard screening-off condition can be considered its special case because if there is no impact at all, there is no negative impact.\(^3\)

These conditions for transitivity are useful,\(^4\) but they are applicable only to simple cases where there is just one step of mediation by \( Y \) and just two possibilities \( Y = \{y, \neg y\} \) to consider. The narrower objective of this paper is to identify the conditions for transitivity in cases with more steps of mediation and more propositions to consider in each step. Achieving this objective requires a more thorough analysis of the mediated probabilistic relation than afforded by (1) and (2). Thus, the broader objective of the paper is to uncover the components and structure of the mediated probabilistic relation.

Some remarks are in order on differences between our undertaking and that of Bayesian networks.\(^5\) The focus of our analysis is the probabilistic relation, which we can often tell without knowing the relevant probabilities. For example, we can often tell that a new piece of evidence raises the probability of a hypothesis even if we are unable to assign any probability (or even a range of probability that is narrow enough for any use) to the hypothesis either unconditionally or on condition of the evidence. The aim of our analysis is therefore different from that of Bayesian networks, which update probabilities on the basis of conditional probabilities assigned to the connected nodes. We will see below that qualitative information on some probabilistic relations is sufficient for determining other probabilistic relations of interest. Apart from having a different focus, our analysis eschews the fundamental assumption of Bayesian networks, namely, the Markov condition that any node in the network must be independent of its non-descendents, given the values of its parent nodes. As we will see shortly, our analysis does not assume that the mediating propositions screen off the supporting proposition from the supported proposition, and this means that the Markov condition need not hold. In the course of analysis we will note some specific points of contrast with Bayesian networks that result from these differences.

2. The Structure of the Mediated Probabilistic Relation

The formulas (1) and (2) serve their respective purposes of establishing \( \text{TC}^> \) and \( \text{TC}^< \) well, but they are not ideal in two ways even in simple cases with one step of mediation and two propositions to consider.

First, they are established on the assumption that the screening-off condition holds for (1) or that its weaker variant holds for (2). The formulas are silent on numerous cases where these conditions do not hold. It is desirable for a full analysis of the mediated probabilistic relation between \( x \) and \( z \) to capture any and every impact \( x \) may have on \( z \), so that the principles of

\(^3\) This means that we need not mention \( \text{TC}^> \) so long as we have \( \text{TC}^< \). However, we keep \( \text{TC}^> \) (and will introduce other principles that are also based on the screening-off condition) in the mix for a few reasons. One is that the screening-off condition is the ‘default’ condition. This will become clearer in the course of our analysis. Also, when we derive a negative result in Section 6, we show the claim fails to hold even under the stronger condition, which is the screening-off condition. Finally, when we generalize our analysis vertically in Section 7, we find a straightforward extension of \( \text{TC}^> \) but not of \( \text{TC}^< \).

\(^4\) Shogenji ([2003]) applies \( \text{TC}^< \) to the epistemological controversy over the testimony of a miracle; Roche and Shogenji ([2014a]) apply \( \text{TC}^< \) to Moore’s proof of an external world.

\(^5\) See (Neapolitan [2004]) and (Pearl [2009]) for standard accounts of Bayesian networks with technical details.
transitivity such as $TC^e$ and $TC^c$ flow naturally from the analysis when special conditions are added.

The second shortcoming is an asymmetry between $y$ and $\neg y$. When the truth of $x$ affects the probability of $y$, it also affects the probability of $\neg y$, which in turn affects the probability of $z$. This part of mediated probabilistic relation is not explicitly represented in (1) and (2). Of course, the change in the probability of $\neg y$ due to $x$ is captured in substance by $Pr(y \mid x) - Pr(y)$ since $Pr(\neg y \mid x) - Pr(\neg y)$ amounts to $-[Pr(y \mid x) - Pr(y)]$. It is not ideal, however, that only the change in the probability of $y$ is explicitly represented in the formulas. In fact the asymmetry causes a problem when we try to generalize them from mediation by $Y = \{y, \neg y\}$ to mediation by $Y = \{y_1, \ldots, y_n\}$. Unlike the simple case, the change in the probability of a single member $y_i$ does not fully determine how the probability of each of the other members changes. As a result, there is no straightforward extension of (1) or (2) to mediation by $Y = \{y_1, \ldots, y_n\}$.

In light of these two shortcomings of (1) and (2), we analyze the mediated probabilistic relation in simple cases in the form of (3) below (proof in Appendix A).

$$Pr(z \mid x) - Pr(z)$$
$$= [Pr(z \mid y) - Pr(z)]Pr(y \mid x) - Pr(y)] + [Pr(z \mid \neg y) - Pr(z)]Pr(\neg y \mid x) - Pr(\neg y)]$$
$$+ Pr(y \mid x)[Pr(z \mid y \land x) - Pr(z \mid y)] + Pr(\neg y \mid x)[Pr(z \mid \neg y \land x) - Pr(z \mid \neg y)]$$

(3) Unlike (1) and (2), the formula (3) is not subject to any restrictions, such as the screening-off condition. To make it easier to see what each component of (3) represents, we rewrite (3) as (4) below with two abbreviations, $C_D(h; e) =_{def} Pr(h \mid e) - Pr(h)$ and $C_D(h; e \mid b) =_{def} Pr(h \mid e \land b) - Pr(h \mid b)$.

$$C_D(z; x) = C_D(z; y)C_D(y; x) + C_D(z; \neg y)C_D(\neg y; x)$$
$$+ Pr(y \mid x)C_D(z; x \mid y) + Pr(\neg y \mid x)C_D(z; x \mid \neg y)$$

(4) $C_D(h; e)$ measures the amount of impact $e$ has on $h$ by the difference between $Pr(h \mid e)$ and $Pr(h)$, while $C_D(h; e \mid b)$ measures the amount of impact $e$ has on $h$ given $b$ by the difference between $Pr(h \mid e \land b)$ and $Pr(h \mid b)$.

Of the four addends of (4) the first two measure the amounts of the indirect impact $x$ has on $z$ through $y$ and through $\neg y$, respectively. Their sum is the indirect impact $x$ has on $z$ through $Y = \{y, \neg y\}$, and is equivalent to the right side of (1) and (2), but since the indirect impact through $\neg y$ is explicitly represented by the second addend, there is a nice symmetry between $y$ and $\neg y$.

The rest of the formula (4) captures the amount of the non-indirect impact $x$ has on $z$. One might say that any impact that is not indirect is direct, but calling this impact ‘direct’ is misleading. The direct impact $x$ has on $z$ in the most natural sense is $C_D(z; x) = Pr(z \mid x) - Pr(z)$, while the non-indirect impact $x$ has on $z$ is the residue of subtracting the indirect impact from $C_D(z; x)$. We call it the ‘bypass’ impact, or the ‘Y-bypass’ impact in the present case. The $Y$-

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6 The variables $h, e, b$ are typically the hypothesis, the evidence, and the background assumptions, respectively. Various measures of confirmation have been proposed in the literature, and some claims about confirmation are measure sensitive, that is, some claims hold when we use one measure but not when we use another (Fitelson 1999). Fortunately, measure sensitivity is not an issue in this paper since the kind of qualitative claims we make below—whether $x$ confirms $z$, disconfirms $z$, or is neutral about $z$—hold regardless of the choice of a measure.

7 Their equivalence can be verified by Bayes’ Theorem.
bypass impact also has two components, the \( y \)-bypass impact \( C_D(z; x \mid y) \) and the \( \neg y \)-bypass impact \( C_D(z; x \mid \neg y) \).

As seen readily, \( x \) has no \( Y \)-bypass impact on \( z \) when \( Y = \{ y, \neg y \} \) screens off \( x \) from \( z \) because if \( \Pr(z \mid y \land x) = \Pr(z \mid y) \) and \( \Pr(z \mid \neg y \land x) = \Pr(z \mid \neg y) \), then \( C_D(z; x \mid y) = 0 \) and \( C_D(z; x \mid \neg y) = 0 \). It should also be noted that the total amount of the \( Y \)-bypass impact \( x \) has on \( z \) is not the sum of the \( y \)-bypass impact and the \( \neg y \)-bypass impact, but their weighted average, where the weights are provided by the conditional probabilities, \( \Pr(y \mid x) \) and \( \Pr(\neg y \mid x) \), respectively.

The formula (4) shows that both the indirect impact through \( Y \) and the \( Y \)-bypass impact have two components each—one associated with \( y \) and the other with \( \neg y \). This allows us to group the four addends of (4) by the mediating proposition, namely, the first and the third addends capture the amount of \( y \)-mediated impact, while the second and the fourth addends capture the amount of \( \neg y \)-mediated impact. The sum of the two mediated impacts is the total impact \( x \) has on \( z \).\(^8\) It is not peculiar to count the \( y \)-bypass and \( \neg y \)-bypass impacts as part of the \( y \)-mediated and \( \neg y \)-mediated impacts, respectively. The amount of the \( y \)-bypass impact \( C_D(z; x \mid y) \) is mediated by \( y \) in the form of the background assumption, that is, it is the amount of impact \( x \) has on \( z \) given the background assumption \( y \). Similarly, the amount of the \( \neg y \)-bypass impact \( C_D(z; x \mid \neg y) \) is mediated by \( \neg y \) in the form of the background assumption \( \neg y \).

So, we can parse the mediated probabilistic relation as follows: When the impact that \( x \) has on \( z \) is mediated by \( Y = \{ y, \neg y \} \), the total impact \( C_D(z; x) \) consists of the \( y \)-mediated impact and \( \neg y \)-mediated impact; the \( y \)-mediated impact in turn consists of the indirect impact \( x \) has on \( z \) through \( y \) and the weighted \( y \)-bypass impact \( x \) has on \( z \); similarly, the \( \neg y \)-mediated impact consists of the indirect impact \( x \) has on \( z \) through \( \neg y \) and the weighted \( \neg y \)-bypass impact \( x \) has on \( z \).

3. Transitivity and Anti-Transitivity

With this analysis in hand we now return to the issue of transitivity. First, our analysis reveals that confirmation is transitive in the indirect part of the mediated probabilistic relation.

\( \text{TC}_1 \) For any propositions \( x, y, z \) if \( x \) confirms \( y \) and \( y \) confirms \( z \), then \( x \) confirms \( z \) through \( y \).

The formal basis of the claim is that if \( C_D(y; x) \) and \( C_D(z; y) \) are positive, then so is \( C_D(y; x)C_D(z; y) \). The principle is not subject to any restrictions, such as the screening-off condition. Of course, it does not follow from this that \( x \) confirms \( z \) overall. Positive indirect impact through \( y \) may be offset by other components of the mediated probabilistic relation. However, it is still significant and reassuring in a way that if \( x \) confirms \( y \) and \( y \) confirms \( z \), then \( x \) confirms \( z \) through \( y \) without exception.

Since \( \text{TC}_1 \) is a general principle, it also applies to the indirect impact \( x \) has on \( z \) through \( \neg y \), that is, if \( C_D(\neg y; x) \) and \( C_D(z; \neg y) \) are positive, then so is \( C_D(\neg y; x)C_D(z; \neg y) \). This is a

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\(^8\) This is in contrast with a Bayesian network. When \( Y = \{ y, \neg y \} \) does not screen off \( x \) from \( z \), the Bayesian network must contain a direct ‘edge’ from \( X = \{ x, \neg x \} \) to \( Z = \{ z, \neg z \} \) in addition to the edge from \( X \) to \( Y \) and the edge from \( Y \) to \( Z \). The additional edge is needed for satisfying the Markov condition.
conditional statement and the antecedent is false in the present case where \( x \) confirms \( y \) and \( y \) confirms \( z \). Instead \( x \) disconfirms \( \neg y \) and \( \neg y \) disconfirms \( z \), so that \( C_D(\neg y; x) \) and \( C_D(z; \neg y) \) are both negative. As it turns out, however, the indirect impact \( x \) has on \( z \) through \( \neg y \) is also positive in the present case because \( C_D(\neg y; x)C_D(z; \neg y) \) is positive when \( C_D(\neg y; x) \) and \( C_D(z; \neg y) \) are both negative.

This brings us to the second consequence of our analysis: disconfirmation is anti-transitive in the indirect part of the mediated probabilistic relation.

**AD\(_1\)** For any propositions \( x, y, z \) if \( x \) disconfirms \( y \) and \( y \) disconfirms \( z \), then \( x \) confirms \( z \) through \( y \).

This principle also holds without any restrictions. In the present case where \( x \) confirms \( y \) and \( y \) confirms \( z \), AD\(_1\) applies to the indirect impact through \( \neg y \), that is, since \( x \) disconfirms \( \neg y \) and \( \neg y \) disconfirms \( z \), \( x \) confirms \( z \) through \( \neg y \). The principle is sensible: If \( x \) reduces the probability of a proposition that disconfirms \( z \), then \( x \) should indirectly raise the probability of \( z \) through its negative impact on the \( \neg y \)-disconfirmation proposition. Of course, the indirect confirmation by AD\(_1\) may be offset by other components of the mediated probabilistic relation, but that does not undermine AD\(_1\) itself.

We can combine TC\(_1\) and AD\(_1\) above into one principle C\(_1\) on the necessary and sufficient condition for positive indirect confirmation.

**C\(_1\)** For any propositions \( x, y, z; x \) confirms \( z \) through \( y \) if and only if either \( x \) confirms \( y \) and \( y \) confirms \( z \), or \( x \) disconfirms \( y \) and \( y \) disconfirms \( z \).

The principle follows directly from the observation that \( C_D(z; y)C_D(y; x) \) is the amount of the indirect impact \( x \) has on \( z \) through \( y \). Two more principles, D\(_1\) and N\(_1\) below, follow from the same observation.

**D\(_1\)** For any propositions \( x, y, z; x \) disconfirms \( z \) through \( y \) if and only if either \( x \) confirms \( y \) and \( y \) disconfirms \( z \), or \( x \) disconfirms \( y \) and \( y \) confirms \( z \).

**N\(_1\)** For any propositions \( x, y, z; x \) neither confirms nor disconfirms \( z \) through \( y \) if and only if either \( x \) neither confirms nor disconfirms \( y \), or \( y \) neither confirms nor disconfirms \( z \).

In combination the three principles classify the indirect impact into three kinds—confirmation by C\(_1\), disconfirmation by D\(_1\), or neutrality by N\(_1\).

In retrospect, principles of transitivity TC\(^=\) and TC\(^≤\) discussed in Section 1 are supported by the combination of transitivity and anti-transitivity. If \( x \) confirms \( y \) and \( y \) confirms \( z \), and thus \( x \) disconfirms \( \neg y \) and \( \neg y \) disconfirms \( z \), then the indirect impact through \( y \) is positive by the transitivity of confirmation, while the indirect impact through \( \neg y \) is positive by the anti-transitivity of disconfirmation. Together they ensure that the indirect impact \( x \) has on \( z \) through \( Y = \{y, \neg y\} \) is positive. It follows immediately that the total impact \( x \) has on \( z \) is positive when the \( Y\)-bypass is zero by the screening-off condition or non-negative by the negative impact screening-off condition.

It is clear from the symmetry of \( y \) and \( \neg y \) in (4) that the outcome remains the same when the roles of \( y \) and \( \neg y \) are reversed, that is, if \( x \) disconfirms \( y \) and \( y \) disconfirms \( z \), and thus \( x \)
confirms $\neg y$ and $\neg y$ confirms $z$, then indirect confirmation through $y$ is positive by the anti-transitivity of disconfirmation, while indirect confirmation through $\neg y$ is positive by the transitivity of confirmation. So, using the same reasoning used in support of $TC^=\wedge TC^\leq$ above, we obtain the following principles of anti-transitivity of disconfirmation.

\begin{align*}
AD^= & \quad \text{For any propositions } x, y, z \text{ if } x \text{ disconfirms } y \text{ and } y \text{ disconfirms } z, \text{ then } x \text{ confirms } z, \text{ provided } Y = \{y, \neg y\} \text{ screens off } x \text{ from } z. \\
AD^\leq & \quad \text{For any propositions } x, y, z \text{ if } x \text{ disconfirms } y \text{ and } y \text{ disconfirms } z, \text{ then } x \text{ confirms } z, \text{ provided } Y = \{y, \neg y\} \text{ screens off the negative impact of } x \text{ from } z.
\end{align*}

$TC^= \wedge AD^= \wedge TC^\leq \wedge AD^\leq$ (and similarly $TC^< \wedge AD^< \wedge TC^\leq \wedge AD^\leq \wedge TC^= \wedge AD^= \wedge TC^\leq \wedge AD^\leq$) are two sides of the same coin. Both are supported by the transitivity of confirmation and the anti-transitivity of disconfirmation. To state the point of $TC^= \wedge AD^= \wedge TC^\leq \wedge AD^\leq$ in neutral terms, $x$ confirms $z$ if the truth of $x$ shifts the probability away from a $z$-disconfirming proposition to a $z$-confirming proposition, provided there is no (negative) bypass impact.

4. Bypass Disconfirmation

So far, our principles are either on the indirect impact ($TC_1, AD_1, C_1, D_1, N_1$) or hold under the screening-off condition or its weaker variant ($TC^=, TC^\leq, AD^=, AD^\leq$) that disallows the bypass impact from offsetting the positive indirect impact. We now turn our focus on cases where the bypass impact offsets the positive indirect impact. We take up two issues in particular from the recent literature. One is the failure of transmission through logical entailment and the other is scepticism about the role of the screening-off condition.

The failure of transmission under logical entailment (‘transmission failure’ for short) occurs when $x$ confirms $y$ and $y$ logically entails $z$, but $x$ does not confirm $z$.\footnote{The issue of transmission failure was brought to light by Wright ([2002], [2004]) and analyzed probabilistically by Okasha ([2004]), Chandler ([2010]), and Moretti ([2012]) among others. The example that follows is due to White ([2006]).} Here is an example. Let us assume, quite plausibly, that the proposition $x$ that it appears to me this is a hand confirms (raises the probability of) the proposition $y$ that this is a hand. On the assumption that a fake hand also appears to be a hand, $x$ confirms the proposition (which we call $\neg z$) that this is a fake hand, while disconfirming $z$. This is not unusual—the evidence often confirms incompatible hypotheses, such as $y$ and $\neg z$, while disconfirming many others, such as that this is a foot, that this is a fake foot, and that this is a dog. What makes this case interesting is that $y$ logically entails the proposition $z$ that this is not a fake hand, so that although $x$ confirms $y$ and $y$ logically entails $z$, $x$ disconfirms $z$.

Cases of transmission failure are also cases against transitivity of confirmation because given the assumption that $0 < \Pr(z) < 1$, if $y$ logically entails $z$ and thus $\Pr(z \mid y) = 1$, then $y$ confirms $z$. So, in those cases of transmission failure, $x$ confirms $y$ and $y$ confirms $z$, but $x$ does not confirm $z$. Transmission failure is of special interest because $y$ not only confirms $z$ but it logically entails $z$. However, it is still helpful to see those cases in terms of confirmation. Recall that indirect confirmation is transitive and indirect disconfirmation is anti-transitive without
exception. This means that even in cases of ‘transmission failure’ $x$ confirms $z$ indirectly—through $y$ by the transitivity of confirmation and through $\neg y$ by the anti-transitivity of disconfirmation. If $x$ does not confirm $z$ overall, it is not because the probabilistic support that $y$ receives from $x$ fails to transmit to $z$ through logical entailment. It is because the positive indirect impact is offset by the negative bypass impact. The term ‘transmission failure’ is unfortunate.

We can be more specific. In the hand case above it is the strong $\neg y$-bypass disconfirmation, $C_D(z; x | \neg y)$, that offsets the positive indirect impact $x$ has on $z$. Given the background assumption $\neg y$ that this is not a hand, the proposition $x$ that it appears to me this is a hand markedly raises the probability of $\neg z$ that this is a fake hand. As a result, $x$ strongly disconfirms $z$ given the background assumption $\neg y$. Meanwhile, $x$ has no $y$-bypass impact on $z$ because the background assumption $y$ (that this is a hand) alone establishes the truth of $z$ (that this is not a fake hand), thereby making $x$ irrelevant to the probability of $z$. It is an advantage of our analysis that it captures all components of the mediated probabilistic relation so that we can see more clearly the component that is primarily responsible for the overall impact.

The same point applies to scepticism about the role of the screening-off condition. Atkinson and Peijnenburg ([2013]) express concern over the screening-off condition used in the principle $TC^z$. The reason for their concern is not $TC^z$ itself, where the role of the screening-off condition is to ensure the transitivity of confirmation, but that the screening-off condition also plays the role of ensuring the anti-transitivity of disconfirmation in $AD^z$. They are concerned that so many cases exist where disconfirmation is not anti-transitive, that is, $x$ disconfirms $y$ and $y$ disconfirms $z$, and $x$ also disconfirms $z$.

Here is one example offered by Atkinson and Peijnenburg. Consider three pairwise incompatible hypotheses $T$, $T^*$ and $T^{**}$. A new piece of evidence $E$ confirms $T^{**}$ while disconfirming $T^*$ and $T$. This is not an uncommon case. Their focus is on the evidence $E$ and the two disconfirmed hypotheses $T^*$ and $T$, which play the roles of $x$, $y$ and $z$, respectively. Since $T^*$ and $T$ are incompatible, $T^*$ disconfirms $T$. But then $E$ disconfirms $T^*$ and $T^*$ in turn disconfirms $T$, and yet $E$ disconfirms $T$ in an apparent violation of the anti-transitivity of disconfirmation.

Of course, the principle $AD^z$ of the anti-transitivity of disconfirmation is derived mathematically from the screening-off condition. So, the abundance of cases against the anti-transitivity of disconfirmation only means that the screening-off condition frequently fails. There is nothing wrong about the principle $AD^z$ itself. But Atkinson and Peijnenburg’s real concern is that the screening-off condition is of little use because the abundance of cases against the anti-transitivity of disconfirmation tells us it is an exception, rather than a norm, that the screening-off condition holds. They want a condition for the transitivity of confirmation that is applicable to a

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10 The point here is qualitative. Since $C_D(y; x)C_D(z; y) < C_D(y; x)$ when $0 < C_D(y; x)$, $C_D(z; y) < 1$, the quantity of indirect confirmation that $x$ provides for $z$ through $y$ is smaller than the quantity of confirmation that $x$ provides for $y$. See (Roche and Shogenji [2014b]) for an analysis of ‘dwindling confirmation’.

11 We noted earlier that a Bayesian network contains a direct ‘edge’ from $X = \{x, \neg x\}$ to $Z = \{z, \neg z\}$ when $Y = \{y, \neg y\}$ does not screen off $x$ from $z$; but the added edge does not make it easier to see the overall impact $x$ has on $z$ because the direct edge only indicates that $X$ is one of $Z$’s parent nodes on which to conditionalize the probabilities. In general a Bayesian network contains no explicit representations of probabilistic impacts, such as confirmation and disconfirmation. Any such relation must be determined from many conditional probabilities.

12 Atkinson and Peijnenburg use the term ‘intransitivity’ instead of ‘anti-transitivity’. Although some authors mean non-transitivity by ‘intransitivity’, it is clear that Atkinson and Peijnenburg mean anti-transitivity.
wider range of cases, more specifically, a condition that does not make disconfirmation anti-transitive.\footnote{Atkinson and Peijnenburg do not discuss Roche’s negative impact screening-off condition, which is weaker than the regular screening-off condition and thus holds in more cases. However, it is of no use for their purpose because disconfirmation is anti-transitive under the weaker condition as well by the principle AD\textsuperscript{5}. They are looking for a condition that does not make disconfirmation anti-transitive.}

Atkinson and Peijnenburg’s own proposal is the partial screening-off condition that only requires the absence of the y-mediated bypass impact, that is, $C_D(z; x \mid y) = 0$ with no restriction on the $-y$-bypass impact $C_D(z; x \mid -y)$. There are many cases, including the one above, that do not satisfy the screening-off condition but satisfy the partial screening-off condition. More to the point, disconfirmation is not anti-transitive under the partial screening-off condition. However, it comes with a price: confirmation is not transitive under the partial screening-off condition, either. Atkinson and Peijnenburg are well aware of this and distinguish two subdomains of probability values under the partial screening-off condition: confirmation is transitive only in one of them.

This is not a coincidence. As discussed already, $x$ confirms $y$ and $y$ confirms $z$ if and only if $x$ disconfirms $-y$ and $-y$ disconfirms $z$. Whichever way we state the condition—the left side or the right side of the biconditional—$x$ shifts the probability away from the $z$-disconfirming proposition to the $z$-confirming proposition. This means that confirmation is transitive so that $x$ confirms $z$ under the left side of the biconditional, if and only if disconfirmation is anti-transitive so that $x$ confirms $z$ under the right side of the biconditional. There is no way of ensuring the transitivity of confirmation without also ensuring the anti-transitivity of disconfirmation.

Whether a certain condition is normal or exceptional is often a matter of a judgment call, but our analysis reveals a clear sense in which the screening-off condition is the ‘default’ condition. Disconfirmation is anti-transitive without exception in the indirect part of the mediated probabilistic relation. Even in their counter-example above—where $E$ disconfirms $T^*$, $T^*$ disconfirms $T$, and $E$ also disconfirms $T$—it remains true that $E$ confirms $T$ through $\{T^*, -T^*\}$ because $E$ shifts the probability away from the $T$-disconfirming proposition $T^*$ to the $T$-confirming proposition $-T^*$. It is only because the strongly negative $-T^*$-bypass impact $C_D(T; E \mid -T^*)$ more than offsets the positive indirect impact that $E$ disconfirms $T$ overall.

In general, and by default, disconfirmation is anti-transitive in the absence of an offsetting impact. It is important to be aware that there is often an offsetting impact in the form of strong bypass disconfirmation, but it is just as important to know that the screening-off condition is the default condition, and that confirmation is transitive and disconfirmation is anti-transitive by default.

5. Horizontal Generalization

We now begin to generalize our analysis. The generalization proceeds in two stages. In this section we generalize the analysis horizontally from mediation by the two-member partition $Y = \{y, -y\}$ to mediation by the $n$-member partition $Y = \{y_1, \ldots, y_n\}$. In Section 7 we generalize it further, vertically, from one step of mediation by $Y$ to $m$ steps of mediation by $Y_1, \ldots, Y_m$.

The setting in this section is as follows. The truth of the proposition $x$ can affect the probability distribution over the partition of propositions $Y = \{y_1, \ldots, y_n\}$.\footnote{One of the $n$ members may be the catch-all (‘none of the above’) hypothesis. $Y = \{y, -y\}$ is a special case where $n = 2$ and $y_2$ is the catch-all hypothesis. We will consider and reject the proposal in the next section that the horizontal
exactly one of its members is true, and depending on which member is true, the probability of the proposition \( z \) may rise, fall, or stay the same. So, the truth of \( x \) can affect the probability of \( z \) indirectly through its impact on the probability distribution over \( Y \). In addition to the indirect impact through \( Y = \{ y_1, \ldots, y_n \} \), \( x \) may also have some \( Y \)-bypass impact. We examine the components and structure of the mediated probabilistic relation in this setting. We will also investigate whether there are principles of transitivity and anti-transitivity similar to those found in Section 3 that are applicable to the new setting.

Since the formula (4) for the case of mediation by \( Y = \{ y, \neg y \} \) is symmetrical between \( y \) and \( \neg y \), we can generalize it straightforwardly to (5) below for mediation by \( Y = \{ y_1, \ldots, y_n \} \) (proof in Appendix B).

\[
C_D(z; x) = \sum_{i=1}^n C_D(z; y_i)C_D(y_i; x) + \sum_{i=1}^n \Pr(y_i \mid x)C_D(z; x \mid y_i)
\] (5)

When \( n = 2 \), we can rewrite \( y_1 \) and \( y_2 \) as \( y \) and \( \neg y \), respectively, to obtain (4) of Section 2 from (5). When \( n \) is 3 or greater, the formula (5) has more components than (4)—each of the two parts of (5) has \( n \) addends—but the basic structure remains the same.

The first part of (5) represents the indirect impact \( x \) has on \( z \) through \( Y = \{ y_1, \ldots, y_n \} \). It has \( n \) addends since there are \( n \) paths of indirect impact—through \( y_1 \), through \( y_2 \), \ldots, and through \( y_n \). Since each of the \( n \) addends is of the form \( C_D(z; y_i)C_D(y_i; x) \), the classification of the indirect impact in Section 3 by \( C_1 \) (confirmation), \( D_1 \) (disconfirmation), and \( N_1 \) (neutrality) carries over.

\[
\begin{align*}
C_1 & \text{ For any propositions } x, y_i, z; \text{ } x \text{ confirms } z \text{ through } y_i \text{ if and only if either } x \text{ confirms } y_i \text{ and } y_i \text{ confirms } z, \text{ or } x \text{ disconfirms } y_i \text{ and } y_i \text{ disconfirms } z. \\
D_1 & \text{ For any propositions } x, y_i, z; \text{ } x \text{ disconfirms } z \text{ through } y_i \text{ if and only if either } x \text{ confirms } y_i \text{ and } y_i \text{ disconfirms } z, \text{ or } x \text{ disconfirms } y_i \text{ and } y_i \text{ confirms } z. \\
N_1 & \text{ For any propositions } x, y_i, z; \text{ } x \text{ neither confirms nor disconfirms } z \text{ through } y_i \text{ if and only if either } x \text{ neither confirms nor disconfirms } y_i \text{, or } y_i \text{ neither confirms nor disconfirms } z.
\end{align*}
\]

The second part of (5) represents the \( Y \)-bypass impact \( x \) has on \( z \). This part also has \( n \) addends, each of which is of the form \( \Pr(y_i \mid x)C_D(z; x \mid y_i) \), where \( C_D(z; x \mid y_i) \) represents the amount of the \( y_i \)-bypass impact \( x \) has on \( z \). It is the amount of impact \( x \) has on \( z \) given the background assumption \( y_i \). As before, the total \( Y \)-bypass impact is not the sum of the \( y_i \)-bypass impacts, but their weighed average, where the weights are provided by the conditional probabilities, \( \Pr(y_i \mid x) \). The \( y_i \)-bypass impacts disappear when \( Y = \{ y_1, \ldots, y_n \} \) screens off \( x \) from \( z \), that is, when \( \Pr(z \mid y_i \land x) = \Pr(z \mid y_i) \) for each \( i \) and thus \( C_D(z; x \mid y_i) = \Pr(z \mid y_i \land x) - \Pr(z \mid y_i) = 0 \) for each \( i \).

If we group the components of (5) by the mediating proposition, the mediated probabilistic relation in the new setting has the following structure: When the impact that \( x \) has on \( z \) is mediated by \( Y = \{ y_1, \ldots, y_n \} \), the total impact \( C_D(z; x) \) consists of \( n \) mediated impacts, that is, the \( y_1 \)-mediated impact, the \( y_2 \)-mediated impact, \ldots, and the \( y_n \)-mediated impact; each \( y_i \)-mediated impact is unnecessary because we can always regard mediation by the \( n \)-member partition \( \{ y_1, \ldots, y_n \} \) as mediation by the two-member partition \( \{ y_i, \neg y_i \} \), or \( \{ y_n, y_1 \lor \ldots \lor y_{i-1} \lor y_{i+1} \lor \ldots \lor y_n \} \).
mediated impact consists of the indirect impact $x$ has on $z$ through $y_i$ and the weighted $y_i$-bypass impact $x$ has on $z$.

So far, everything looks the same as before, but differences emerge when we look at the total impact $x$ has on $z$. In simple cases of mediation by $Y = \{y, \neg y\}$, the principle $TC^-$ states that confirmation is transitive by default, that is, if $x$ confirms $y$ and $y$ confirms $z$, then $x$ confirms $z$, provided there is no $Y$-bypass impact (provided $Y$ screens off $x$ from $z$). We acknowledged there that $x$ has two indirect impacts on $z$ through $y$ and through $\neg y$, and $x$ actually disconfirms $\neg y$ and $\neg y$ disconfirms $z$ when $x$ confirms $y$ and $y$ confirms $z$. However, $x$ still confirms $z$ through $\neg y$ because of the anti-transitivity of disconfirmation. As a result, $x$ confirms $z$ by default (in the absence of a $Y$-bypass impact) because $x$ indirectly confirms $z$ both through $y$ and through $\neg y$.

This neat reasoning does not hold in cases of mediation by $Y = \{y_1, \ldots, y_n\}$. For illustration, suppose the probabilistic relation is mediated by $Y = \{y_1, y_2, y_3\}$. If $x$ confirms $y_1$ and $y_1$ confirms $z$, then $x$ confirms $z$ through $y_1$. However, we cannot tell whether $x$ confirms $y_2$, disconfirms $y_2$, or is neutral; nor can we tell whether $y_2$ confirms $z$, disconfirms $z$, or is neutral. That is also true of $y_3$. We cannot tell whether $x$ confirms $y_3$, disconfirms $y_3$, or is neutral; and we cannot tell whether $y_3$ confirms $z$, disconfirms $z$, or is neutral. So, it is possible, for example, that $x$ confirms $y_2$ and $y_2$ disconfirms $z$, so that $x$ disconfirms $z$ through $y_2$ by the principle $D_1$, while $x$ disconfirms $y_3$ and $y_3$ confirms $z$, so that $x$ disconfirms $z$ through $y_3$ also by the principle $D_1$. There is therefore no guarantee that $x$ confirms $z$ overall even if there is no $Y$-bypass impact.

One way to ensure that $x$ confirms $z$ overall by default (under the screening-off condition) is to require that $x$ confirm all $z$-confirming $y_i$'s and disconfirm all $z$-disconfirming $y_i$'s. This makes the indirect impact positive through each of the $n$ paths. But it is not necessary for overall confirmation by default that the indirect impact be positive through each of the $n$ paths. We can weaken the requirement, namely, the indirect impact is positive at least through one path, and non-negative though any path.

\[ C_n^- \] For any propositions $x$ and $z$, and any partition \( \{y_1, \ldots, y_n\} \) of propositions, if for some $i$ either \( C_D(y_i; x) > 0 \) and \( C_D(z; y_i) > 0 \), or \( C_D(y_i; x) < 0 \) and \( C_D(z; y_i) < 0 \), and for no $i$ either \( C_D(y_i; x) < 0 \) and \( C_D(z; y_i) > 0 \), or \( C_D(y_i; x) > 0 \) and \( C_D(z; y_i) < 0 \), then \( C_D(z; x) = 0 \), provided \( \{y_1, \ldots, y_n\} \) screens off $x$ from $z$.\(^\text{15}\)

This is the general principle of overall confirmation by default (under the screening-off condition), but there are two special cases where we only need to know the indirect impact through each of $n-1$ paths instead of each of $n$ paths.\(^\text{16}\)

Suppose in each of the $n-1$ paths $x$ confirms $y_i$ and $y_i$ confirms $z$. It follows by the transitivity of indirect confirmation that $x$ confirms $z$ through $y_i$ in each of the $n-1$ paths. Moreover, the condition ensures that $x$ disconfirms $y_i$ and $y_i$ disconfirms $z$ in the remaining one path because if $x$ raises the probabilities of some members of $Y$, it must reduce the probability of at least one member of $Y$; and similarly if some members of $Y$ raise the probability of $z$, at least one member of $Y$ must reduce the probability of $z$. So, $x$ confirms $z$ through the remaining one path by the anti-transitivity of disconfirmation. Since $x$ confirms $z$ through $y_i$ for each of the $n$ paths, $x$ confirms $z$ overall by default (under the screening-off condition).

\(^\text{15}\) We can weaken the requirement in $C_n^-$ further by replacing the screening-off condition by the negative impact screening-off condition.

\(^\text{16}\) The mediating proposition for the remaining one path may be the catch-all (‘none of the above’) hypothesis.
We can also weaken the requirement in the special case because it is not necessary for overall confirmation by default that the indirect impact be positive through each of the \( n \) paths. We only require in the special case as well that the indirect impact be positive at least through one path, and non-negative though any path, as follows.

\[
TC_n^= \text{For any propositions } x \text{ and } z, \text{ and any partition } \{ y_1, \ldots, y_n \} \text{ of propositions, if } C_D(y_i; x) > 0 \text{ and } C_D(z; y_i) > 0 \text{ for some } i, \text{ and for any } i \text{ except one } C_D(y_i; x) \geq 0 \text{ and } C_D(z; y_i) \geq 0, \text{ then } C_D(z; x) > 0, \text{ provided } \{ y_1, \ldots, y_n \} \text{ screens off } x \text{ from } z.
\]

This is a generalization of the principle \( TC^= \) in Section 1. By similar reasoning we can also generalize \( AD^= \) in Section 3 as follows.\(^{17}\)

\[
AD_n^= \text{For any propositions } x \text{ and } z, \text{ and any partition } \{ y_1, \ldots, y_n \} \text{ of propositions, if } C_D(y_i; x) < 0 \text{ and } C_D(z; y_i) < 0 \text{ for some } i, \text{ and for any } i \text{ except one } C_D(y_i; x) \leq 0 \text{ and } C_D(z; y_i) \leq 0, \text{ then } C_D(z; x) > 0, \text{ provided } \{ y_1, \ldots, y_n \} \text{ screens off } x \text{ from } z.
\]

6. Coarse Screens

This section spells out some details of the screening-off condition. We also see in the course of the discussion how the horizontal generalization of Section 5 helps us determine the overall impact of new information in the mediated probabilistic relation.

The key point in the reasoning for the two principle \( TC_n^= \) of Section 5 is that \( x \) cannot confirm every \( y_i \) in the partition, and that not every \( y_i \) in the partition can confirm \( z \). It may seem that we can use this point in a more straightforward way to bring back the original principle \( TC^= \) even in cases of mediation by \( Y = \{ y_1, \ldots, y_n \} \). Suppose \( x \) confirms \( y_i \) and \( y_i \) confirms \( z \) for some member \( y_i \) of the partition \( Y = \{ y_1, \ldots, y_n \} \). It follows by the principle \( C_1 \) that \( x \) confirms \( z \) through \( y_i \). It also follows from the supposition that \( x \) disconfirms \( \neg y_i \) and \( \neg y_i \) disconfirms \( z \). So, \( x \) confirms \( z \) through \( \neg y_i \) by the principle \( C_1 \) as well. Since \( x \) confirms \( z \) both through \( y_i \) and through \( \neg y_i \), we can conclude, it seems, that \( x \) confirms \( z \) by default when \( Y \) screens off \( x \) from \( z \).

According to this reasoning, it does not matter if \( x \) disconfirms \( z \) through some members of \( Y = \{ y_1, \ldots, y_n \} \). For example, it is possible that \( x \) confirms \( y_j \) and \( y_j \) disconfirms \( z \) for some \( j \neq i \), so that \( x \) disconfirms \( z \) through \( y_j \). However, by the reasoning above \( x \) confirms \( z \) through \( \neg y_i \), which is the disjunction \( y_1 \lor \ldots \lor y_{i-1} \lor y_{i+1} \lor \ldots \lor y_n \), and \( y_j \) is one of the disjuncts. This means that the negative impact \( x \) has on \( z \) through \( y_j \) is offset by the positive impacts \( x \) has on \( z \) through some of the other disjuncts because \( x \) confirms \( z \) through the disjunction as a whole. The apparent implication is that we do not need \( TC_n^= \) because \( TC^= \) works fine even in cases of mediation by \( Y = \{ y_1, \ldots, y_n \} \). As long as there is one member \( y_i \) such that \( x \) confirms \( y_i \) and \( y_i \) confirms \( z \), we can take \( y_i \) and \( y_1 \lor \ldots \lor y_{i-1} \lor y_{i+1} \lor \ldots \lor y_n \) to be \( y \) and \( \neg y \) of \( TC^= \), respectively. By the same reasoning, as long as there is one member \( y_j \) such that \( x \) disconfirms \( y_j \) and \( y_j \) disconfirms \( z \), we can take \( y_j \) and \( y_1 \lor \ldots \lor y_{i-1} \lor y_{i+1} \lor \ldots \lor y_n \) to be \( y \) and \( \neg y \) of \( AD^= \), respectively. So, we do not need \( AD_n^= \) either because \( AD^= \) works fine even in cases of mediation by \( Y = \{ y_1, \ldots, y_n \} \), or so it seems.

\(^{17}\)We can weaken the requirements in \( TC_n^= \) and \( AD_n^= \) by replacing the screening-off condition by the negative impact screening-off condition.
Unfortunately, the reasoning is incorrect. We can see it is incorrect by applying the same reasoning to obtain the opposite conclusion. Suppose \( x \) confirms \( y_i \) and \( y_j \) disconfirms \( z \) for some member \( y_j \) of the partition \( Y = \{ y_1, \ldots, y_n \} \). It follows that \( x \) disconfirms \( z \) through \( y_j \) by the principle \( D_1 \). It also follows from the supposition that \( x \) disconfirms \( \neg y_j \) and \( \neg y_j \) confirms \( z \). So, \( x \) also disconfirms \( z \) through \(-y_j\) by the principle \( D_1 \). Since \( x \) disconfirms \( z \) both through \( y_j \) and through \( -y_j \), we can conclude, it seems, that \( x \) disconfirms \( z \) by default when \( Y \) screens off \( x \) from \( z \). The trouble is that it is possible when \( n \) is 3 or greater that (a) \( x \) confirms \( y_i \) and \( y_j \) confirms \( z \) for some member \( y_i \) of the partition \( Y = \{ y_1, \ldots, y_n \} \), while (b) \( x \) confirms \( y_j \) and \( y_j \) disconfirms \( z \) for another member \( y_j \) of the same partition. This means that the same line of reasoning—applied to (a) and (b), respectively—allows us to conclude that \( x \) confirms \( z \) and \( x \) disconfirms \( z \) both by default when \( Y \) screens off \( x \) from \( z \). Something is wrong.

Here is our diagnosis. The reasoning is correct up to a point. For example, if (a) \( x \) confirms \( y_i \) and \( y_j \) confirms \( z \) for some member \( y_i \) of \( Y = \{ y_1, \ldots, y_n \} \), then \( C_D(y_i; x)C_D(z; y_i) > 0 \) and \( C_D(-y_i; x)C_D(z; -y_i) > 0 \), so that the total indirect impact is positive. However, it is the total indirect impact through the two-member partition \{\( y_i, -y_i \)\} that is positive. It does not follow that \( x \) confirms \( z \) by default when \( Y = \{ y_1, \ldots, y_n \} \) screens off \( x \) from \( z \). Similarly, if (b) \( x \) confirms \( y_j \) and \( y_j \) disconfirms \( z \) for another member \( y_j \), then \( C_D(y_j; x)C_D(z; y_j) < 0 \) and \( C_D(-y_j; x)C_D(z; -y_j) < 0 \), so that the total indirect impact is negative. However, it is the total indirect impact through the two-member partition \{\( y_j, -y_j \)\} that is negative. It does not follow that \( x \) disconfirms \( z \) by default when \( Y = \{ y_1, \ldots, y_n \} \) screens off \( x \) from \( z \). Neither (a) nor (b) tells us anything about the total indirect impact through the two-member partition \( Y = \{ y_1, \ldots, y_n \} \). For that, we need the principle \( C_n^* \) (or the special principle \( TC_n^* \) or \( AD_n^* \)) that is intended for mediation by \( Y = \{ y_1, \ldots, y_n \} \).

Some people may think our diagnosis misses the point of the suggestion that we can use \( T^* \) and \( A^* \) instead of \( C_n^* \) (instead of \( T_n^* \) and \( A_n^* \) in special cases). It may be pointed out that if we can tell by \( T^* \) or \( A^* \) that the total indirect impact through \( \{ y_i, -y_i \} \) is positive, then we can conclude that \( x \) confirms \( z \) by default, which in this case means that the two-member partition \{\( y_i, -y_i \)\} screens off \( x \) from \( z \). We need to be aware that we cannot draw the same conclusion from the condition that \( Y = \{ y_1, \ldots, y_n \} \) screens off \( x \) from \( z \), but it only means that \( Y = \{ y_1, \ldots, y_n \} \) is not the right screen in the case. Similarly, if the total indirect impact through \{\( y_j, -y_j \)\} is negative, we can conclude that \( x \) disconfirms \( z \) by default, which in this case means that the two-member partition \{\( y_j, -y_j \)\} screens off \( x \) from \( z \). Again, \( Y = \{ y_1, \ldots, y_n \} \) is not the right screen in the case.

The problem with this suggestion is that coarse screens such as \{\( y_i, -y_i \)\} and \{\( y_j, -y_j \)\} often fail to screen off \( x \) from \( z \) even if \( Y = \{ y_1, \ldots, y_n \} \) screens off \( x \) from \( z \). We can see this from the example above. Given (a) and (b), it is still possible that \( Y = \{ y_1, \ldots, y_n \} \) screens off \( x \) from \( z \). However, given (a) and (b), either \( y_i, -y_i \) or \( y_j, -y_j \) fails to screen off \( x \) from \( z \) because if each of them screens off \( x \) from \( z \), we have the contradiction that \( x \) confirms \( z \) and disconfirms \( z \). Another difficulty is that judging whether \( \{ y_i, -y_i \} = \{ y_i, y_1 \lor \cdots \lor y_{i+1} \lor \cdots \lor y_n \} \) screens off \( x \) from \( z \) is usually much harder than judging whether \( \{ y_1, \ldots, y_n \} \) does. The example below illustrates these two points.

Let \( x \) and \( z \) be the propositions that your new neighbor bought a sports car, and that she is rich, respectively. There are exactly four car dealers \( A, B, C \) and \( D \) in town. So, we let \( Y = \{ y_1, y_2, y_3, y_4 \} \) be a partition, where \( y_1, y_2, y_3, y_4 \) are the propositions that the new neighbor bought her car from dealer \( A \), dealer \( B \), dealer \( C \), and dealer \( D \), respectively. Let us also assume the following about the four dealers. Dealers \( A \) and \( B \) only sell economy cars and carry few sports cars. We assume that \( x \) disconfirms both \( y_1 \) and \( y_2 \), and both \( y_1 \) and \( y_2 \) disconfirm \( z \), so that \( x \)
confirms \( z \) both through \( y_1 \) and through \( y_2 \). But there is a twist. Dealer A targets younger customers while dealer B targets older customers. So, dealer A carries more sports cars in comparison with dealer B, but dealer A tends to sell more inexpensive cars in comparison with dealer A. As a result, though the proposition \( x \) that your new neighbor bought a sports car disconfirms both \( y_1 \) and \( y_2 \), it changes the balance of probabilities between \( y_1 \) and \( y_2 \) in favor of \( y_1 \) in the sense that \( \Pr(y_1)/\Pr(y_2) < \Pr(y_1 \mid x)/\Pr(y_2 \mid x) \). Also, though both \( y_1 \) and \( y_2 \) disconfirm \( z \), \( y_1 \) does so more strongly than \( y_2 \) does. Meanwhile, dealers C and D sell luxury cars and carry more sports cars. We assume that \( x \) confirms both \( y_3 \) and \( y_4 \), and both \( y_3 \) and \( y_4 \) confirm \( z \), so that \( x \) confirms \( z \) both through \( y_3 \) and through \( y_4 \). There is a similar twist between dealers C and D as well. Dealer C targets younger customers while dealer D targets older customers. So, dealer C carries more sports cars in comparison with dealer D, but dealer C tends to sell more inexpensive cars in comparison with dealer D. As a result, though the proposition \( x \) that your new neighbor bought a sports car confirms both \( y_3 \) and \( y_4 \), it changes the balance of probabilities between \( y_3 \) and \( y_4 \) in favor of \( y_3 \) in the sense that \( \Pr(y_3)/\Pr(y_4) < \Pr(y_3 \mid x)/\Pr(y_4 \mid x) \). Also, though both \( y_3 \) and \( y_4 \) confirm \( z \), \( y_3 \) does so less strongly than \( y_4 \) does. Finally, we assume that \( Y = \{y_1, y_2, y_3, y_4\} \) screens off \( x \) from \( z \); in other words, once you identify the dealer from which your new neighbor bought her car, the additional information that she bought a sports car does not affect the probability that she is rich. We can conclude from these assumptions that \( x \) confirms \( z \) by the principle \( C_{=n} \).

The point of this example is that we cannot draw the same conclusion by the principle \( TC_{=n} \) or \( AD_{=n} \). It may seem reasonable to put \( y_1 \) and \( y_2 \) together and \( y_3 \) and \( y_4 \) together to form a two-member partition \( \{y_1 \lor y_2, y_3 \lor y_4\} \). Since \( x \) disconfirms both \( y_1 \) and \( y_2 \) and both of them in turn disconfirm \( z \), \( x \) disconfirms \( y_1 \lor y_2 \) and \( y_1 \lor y_2 \) disconfirms \( z \). It follows by \( AD_1 \) of Section 3 that \( x \) confirms \( z \) through \( y_1 \lor y_2 \). Similarly, since \( x \) confirms both \( y_3 \) and \( y_4 \) and both of them in turn confirm \( z \), \( x \) confirms \( y_3 \lor y_4 \) and \( y_3 \lor y_4 \) confirms \( z \). It follows by \( TC_1 \) of Section 3 that \( x \) confirms \( z \) through \( y_3 \lor y_4 \). By putting them together, we infer that \( x \) confirms \( z \) indirectly through \( \{y_1 \lor y_2, y_3 \lor y_4\} \). However, we still cannot conclude by \( TC_{=n} \) or \( AD_{=n} \) that \( x \) confirms \( z \) overall because \( \{y_1 \lor y_2, y_3 \lor y_4\} \) does not screen off \( x \) from \( z \). For example, even after you come to know that the new neighbor bought her car from either dealer A or dealer B, the additional information \( x \) that she bought a sports car still reduces the probability of \( z \) that she is rich because \( x \) shifts the probability away from \( y_2 \) to \( y_1 \), and \( y_1 \) disconfirms \( z \) more strongly than \( y_2 \) does.\(^{18}\) Similarly, even after you come to know that the new neighbor bought her car from either dealer C or dealer D, the additional information \( x \) that she bought a sports car still reduces the probability of \( z \) that she is rich because \( x \) shifts the probability away from \( y_4 \) to \( y_3 \), and \( y_3 \) does not confirm \( z \) as strongly as \( y_4 \) does.

It does not help to divide the dealers in a different way. For example, if we put dealers B, C and D together, then \( x \) confirms \( y_2 \lor y_3 \lor y_4 \) (since \( x \) disconfirms \( y_1 \)) and \( y_2 \lor y_3 \lor y_4 \) in turn confirms \( z \) (since \( y_1 \) disconfirms \( z \)). We can therefore conclude that \( x \) indirectly confirms \( z \) through \( \{y_1, y_2 \lor y_3 \lor y_4\} \). However, we cannot tell from the information given whether the \( y_2 \lor y_3 \lor y_4 \)-bypass impact is positive, negative, or neutral.\(^{19}\) As a result, we cannot conclude by \( TC_{=n} \)

\[^{18}\text{Note that we cannot use } TC_{=n} \text{ or } AD_{=n} \text{ either in this case. Since } x \text{ disconfirms } z \text{ given the background assumption } y_1 \lor y_2, \text{ the partition } \{y_1 \lor y_2, y_3 \lor y_4\} \text{ does not screen off the negative impact of } x \text{ on } z.\]

\[^{19}\text{This is because given } y_2 \lor y_3 \lor y_4, \text{ the additional information } x \text{ has two impacts on } z \text{ in the opposite directions. First, } x \text{ shifts the probability away from } y_2 \lor y_3 \lor y_4, \text{ thereby raising the probability of } z. \text{ Second, } x \text{ also shifts the probability from } y_4 \lor y_3 \text{, thereby reducing the probability of } z. \text{ We cannot tell from the information given whether}\]
or \( AD^\parallel \) that \( x \) confirms \( z \) overall. Similarly, if we put dealers A, C, and D together, A, B and D together, or A, B and C together, then \( x \) indirectly confirms \( z \) through \( \{ y_2, y_1 \lor y_3 \lor y_4 \} \), through \( \{ y_3, y_1 \lor y_2 \lor y_4 \} \), or through \( \{ y_4, y_1 \lor y_2 \lor y_3 \} \), respectively. However, we cannot tell from the information given whether the disjunction-bypass impact is positive, negative, or neutral in these cases. As a result, we cannot conclude by \( TC^\parallel \) or \( AD^\parallel \) that \( x \) confirms \( z \). There are two other ways of dividing the dealers into two groups, \( \{ y_1 \lor y_3, y_2 \lor y_4 \} \) and \( \{ y_1 \lor y_4, y_2 \lor y_3 \} \). Though neither of them screens of \( x \) from \( z \), each of them actually screens off the negative impact of \( x \) from \( z \). However, neither \( TC^\leq \) nor \( AD^\leq \) is applicable after all because we cannot tell from the information given whether \( x \) indirectly confirms \( z \) through either of these partitions.20

In sum the horizontal generalization of Section 5 is not an idle exercise since the principle \( C_{n}^{\parallel} \) (and the principles \( TC_{n}^{\parallel} \) and \( AD_{n}^{\parallel} \) in special cases) obtained there often plays a crucial role and cannot be replaced by \( TC^{\parallel} \) or \( AD^{\parallel} \). There are two reasons. First, there are cases in which the \( n \)-member partition \( Y = \{ y_1, \ldots, y_n \} \) screens off \( x \) from \( z \), but the coarse screens that group together some of the members into a disjunction(s) fail to do so. Second, it is usually much easier to judge whether \( Y = \{ y_1, \ldots, y_n \} \) screens off \( x \) from \( z \) than to judge whether any of the course screens does.

7. Vertical Generalization

In this section we generalize our analysis of the mediated probabilistic relation vertically for cases involving two or more steps of mediation. The setting is as follows. The probabilistic relation between \( x \) and \( z \) is mediated by \( m \) layers of partitions, \( Y_1, \ldots, Y_m \). The \( j \)-th layer \( Y_j \) has \( n_j \) members, and we call the \( i \)-th member of the \( j \)-th layer \( y_{j,i} \) so that \( Y_1 = \{ y_{1,1}, \ldots, y_{1,n_1} \}, \ldots, Y_m = \{ y_{m,1}, \ldots, y_{m,n_m} \} \). The truth of the proposition \( x \) can affect the probability distribution over the first partition \( Y_1 = \{ y_{1,1}, \ldots, y_{1,n_1} \} \). The truth of a different member of \( Y_1 \) can in turn affect the probability distribution over the next partition \( Y_2 = \{ y_{2,1}, \ldots, y_{2,n_2} \} \) differently. The same holds for any adjacent pair of partitions, that is, the truth of a different member of \( Y_j = \{ y_{j,1}, \ldots, y_{j,n_j} \} \) can affect the probability distribution over the next layer of partition \( Y_{j+1} = \{ y_{j+1,1}, \ldots, y_{j+1,n_{j+1}} \} \) differently. Finally, the truth of a different member of \( Y_m = \{ y_{m,1}, \ldots, y_{m,n_m} \} \) can affect the probability of \( z \) differently. When the probabilistic relation is mediated by \( m \) layers of partitions \( Y_1, \ldots, Y_m \) in this way, there are \( n_1 \times \ldots \times n_m \) different chains of propositions through which \( x \) can affect the probability of \( z \); in addition, the truth of any proposition, except those in the last layer, may also have some bypass impact on \( z \).

If we are only interested in determining whether or not \( x \) confirms \( z \) overall, we may apply the horizontally generalized analysis of Section 5 successively to each layer of partition. In the first step, we determine what \( Y_1 \)-mediated impact \( x \) has on each member of \( Y_2 \). In the second step, based in part on the results of the first step, we determine what \( Y_2 \)-mediated impact \( x \) has on

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20 We can tell from the information given that \( y_1 \) disconfirms \( z \) more strongly than \( y_2 \) does, and that \( y_3 \) does not confirm \( z \) as strongly as \( y_2 \) does, but it does not follow that \( y_1 \lor y_3 \) disconfirms \( z \) while \( y_2 \lor y_4 \) confirms \( z \). That depends further on the probability distribution over \( \{ y_1, y_2, y_3, y_4 \} \).
each member of \( Y_3 \). We continue this process \( m \) times to determine what \( Y_m \)-mediated impact \( x \) has on \( z \). However, this approach is not helpful in uncovering the components and structure of the probabilistic relation in the new setting. For example, it remains unclear by this approach what indirect impact \( x \) has on \( z \) through each of the \( n_1 \times \ldots \times n_m \) chains of propositions that connect them. It is also unclear how the bypass impact is structured overall. We are going to answer these questions in this section.

The following is the probabilistic relation between \( x \) and \( z \) mediated by \( m \) layers of partitions \( Y_1 = \{ y_{1,1}, \ldots, y_{1,n_1} \}, \ldots, Y_m = \{ y_{m,1}, \ldots, y_{m,n_m} \} \) (proof in Appendix C).

\[
\Pr(z \mid x) - \Pr(z) = \sum_{i=1}^{n_m} \sum_{j=1}^{n_j} \Pr(y_{m,i} \mid x) C_D(z; y_{m,i}) \prod_{j=1}^{m-1} C_D(y_{j+1,i}; y_{j,i}) C_D(y_{1,i}; x)
\]

\[
+ \sum_{j=2}^{m} \sum_{j=1}^{n_j} \Pr(y_{j-1,i} \mid y_{j,i}) C_D(z; y_{j-1,i}) \prod_{k=1}^{j-1} C_D(y_{k+1,i}; y_{k,i}) C_D(y_{1,i}; x)
\]

\[
+ \sum_{i=1}^{n_m} \Pr(y_{1,i} \mid x) C_D(z; x \mid y_{1,i})
\]

(6)

The components and their structure are a little easier to see when we rewrite (6) as follows.

\[
\Pr(z \mid x) - \Pr(z) = \sum_{i=1}^{n_m} \sum_{j=1}^{n_j} \Pr(y_{m,i} \mid x) C_D(z; y_{m,i}) \prod_{j=1}^{m-1} C_D(y_{j+1,i}; y_{j,i}) C_D(y_{1,i}; x)
\]

\[
+ \sum_{j=2}^{m} \sum_{j=1}^{n_j} \Pr(y_{j-1,i} \mid y_{j,i}) C_D(z; y_{j-1,i}) \prod_{k=1}^{j-1} C_D(y_{k+1,i}; y_{k,i}) C_D(y_{1,i}; x)
\]

\[
\ldots
\]

\[
+ \sum_{i=1}^{n_m} \Pr(y_{1,i} \mid x) C_D(z; x \mid y_{1,i})
\]

(7)

The first line of (7) represents the purely indirect impact \( x \) has on \( z \) through the \( m \) layers of partitions; the last line of (7) represents the purely bypass impact \( x \) has on \( z \); and the rest (the middle lines) represent the mixed impacts \( x \) has on \( z \).\(^{21}\) Let us take a closer look at each component.\(^{22}\)

The purely indirect impact is a straightforward extension of our previous result. Recall that in cases of mediation by a single layer of partition \( Y = \{ y_1, \ldots, y_n \} \) the indirect impact \( x \) has on \( z \) through each of the \( n \) member of \( Y \) is the product \( C_D(z; y_i) C_D(y_i; x) \) of the two impacts—\( C_D(z; y_i) \) is the impact \( x \) has on \( y_i \) and \( C_D(y_i; x) \) is the impact \( y_i \) has on \( z \). We can think of each of \( n \) paths as a chain of three propositions \( x, y_i, z \) with two links. Where there are \( m \) layers of partitions \( Y_1 = \{ y_{1,1}, \ldots, y_{1,n_1} \}, \ldots, Y_m = \{ y_{m,1}, \ldots, y_{m,n_m} \} \), there are \( n_1 \times \ldots \times n_m \) paths of indirect

\(^{21}\) This is not the only way of parsing the components of the probabilistic relation mediated by the \( m \) layers of partitions, but it is the most illuminating way in our view.

\(^{22}\) That (7) captures the entirety of the impact that \( x \) has on \( z \) is important, especially in contrast with Bayesian networks. Take a Bayesian network in which \( X = \{ x, \neg x \} \) and \( Z = \{ z, \neg z \} \) are multiply connected by two paths, \( [X, Y_1, \ldots, Y_m, Z] \) and \( [X, W_1, \ldots, W_g, Z] \). In order to compute \( \Pr(z \mid x) - \Pr(z) \) from the conditional probabilities alone that are assigned to the nodes in the network, we must put together information from both paths. There is no such need with (7). We can simply apply (7) to one of the paths in disregard of the other—provided, of course, the values of all components of (7) are given—since (7) captures the entirety of the impact that \( x \) has on \( z \).
impact instead of \( n \), but the impact through each path is still determined in the same way, that is, for each chain of propositions \( x, y_{1,i}, \ldots, y_{m,a} \), \( z \) the indirect impact \( x \) has on \( z \) through it is the product \( C_D(z; y_{m,i,a}) C_D( y_{m,i,a}; y_{m-1,i,a-1} ) \cdots C_D( y_{2,i}; y_{1,i}) C_D( y_{1,i} ; x) \) of the impacts in the \( m + 1 \) links of the chain. The total amount of the purely indirect impact \( x \) has on \( z \) through the \( m \) layers of partitions is the sum of the purely indirect impact \( x \) has on \( z \) through each of the \( n_1 \times \ldots \times n_m \) paths.

The formula reveals that we can classify the indirect impact through any of the \( n_1 \times \ldots \times n_m \) paths into three kinds.

\[ C_{Im} \quad \text{For any chain of propositions } x, y_{1,i}, \ldots, y_{m,a} \text{, } z; x \text{ confirms } z \text{ through the chain if and only if the impact in no link in the chain is zero and the number of links in which the impact is negative is zero or even.} \]

\[ D_{Im} \quad \text{For any chain of propositions } x, y_{1,i}, \ldots, y_{m,a} \text{, } z; x \text{ disconfirms } z \text{ through the chain if and only if the impact in no link in the chain is zero and the number of links in which the impact is negative is odd.} \]

\[ N_{Im} \quad \text{For any chain of propositions } x, y_{1,i}, \ldots, y_{m,a} \text{, } z; x \text{ neither confirms nor disconfirms } z \text{ through the chain if and only if there is at least one link in which the impact is zero.} \]

The purely bypass impact represented by the last line of (7) is identical to the bypass impact in mediation by a single layer of partition, namely, the \( y_{1,i} \)-bypass impact \( x \) has on \( z \) is the amount of impact \( C_D(z; y_{1,i}) \) that \( x \) has on \( z \) given \( y_{1,i} \). If \( Y_1 = \{ y_{1,1}, \ldots, y_{1,n_1} \} \) screens off \( x \) from \( z \), then \( x \) has no purely bypass impact on \( z \). As before, the total amount of the \( Y_1 \)-bypass impact \( x \) has on \( z \) is not the sum of the \( y_{1,i} \)-bypass impacts but their weighted average, where the weights are provided by the conditional probabilities, \( \Pr( y_{1,i} \mid x) \).\(^{23}\)

The mixed impact represented by the rest of (7) has a more intricate structure. The bypass part of the mixed impact has a familiar form except that it is a bypass impact that a member of a partition has on \( z \). For example, the \( y_{j,i} \)-bypass impact \( y_{j-1,i-1} \) has on \( z \) is the impact

\[ C_D(z; y_{j-1,i-1} \mid y_{j,i}) \] that \( y_{j-1,i-1} \) has on \( z \) given \( y_{j,i} \). If \( Y_j = \{ y_{j,1}, \ldots, y_{j,n_j} \} \) screens off each member of \( Y_{j-1} = \{ y_{j-1,1}, \ldots, y_{j-1,n_{j-1}} \} \) from \( z \), then no member of \( Y_{j-1} = \{ y_{j-1,1}, \ldots, y_{j-1,n_{j-1}} \} \) has any \( Y_j \)-bypass impact on \( z \). As before, the total amount of the \( Y_j \)-bypass impact that \( y_{j-1,i-1} \) has on \( z \) is not the sum of the \( y_{1,i} \)-bypass impacts but their weighted average, where the weights are provided by the conditional probabilities, \( \Pr( y_{j,i} \mid y_{j-1,i-1}) \). Everything looks the same up to this

\(^{23}\) If one wishes to measure the amount of the \( y_{1,i} \)-bypass impact associated with each chain of propositions that connects \( x \) to \( z \), we can rewrite \( C_D(z; x \mid y_{1,i}) \) as \( \sum_{z_{2;i}} \sum_{y_{2;i}} C_D(z \land y_{m;i} \land \ldots \land y_{1;i} ; x \mid y_{1;i}) \). The amount of the purely bypass impact associated with the particular chain of propositions \( y_{1,i}, \ldots, y_{m,i} \) is then the amount of impact \( C_D(z \land y_{m;i} \land \ldots \land y_{1;i}; x \mid y_{1;i}) \) that \( x \) has on the conjunction \( z \land y_{m;i} \land \ldots \land y_{2;i} \) given \( y_{1,i} \).
point, but the crucial difference is that the weighted \( y_{ij} \)-bypass impact in the mixed impact is further multiplied by \( C_D(y_{j-1,i_j}; y_{j-1,i_{j+1}}) \cdots C_D(y_{1,i_1}; x) \). The resulting product captures the amount of the mixed impact in the following sense; it represents the impact \( x \) has on \( z \) through the partial chain of propositions from \( x \) up to \( y_{j-1,i_{j+1}} \) and then via the \( y_{j,i_j} \)-bypass that connects \( y_{j-1,i_{j+1}} \) to \( z \) with no intermediary propositions.

The presence of the mixed impact has some practical consequences. We noted earlier in Sections 1 and 5 that the screening-off condition required in the principles \( TC^\geq \) and \( TC^\leq_n \) can be weakened to the negative impact screening-off condition in the principles \( TC^\leq \) and \( TC^-n \). This is because only the positive bypass impact can offset the positive indirect impact, but this is no longer true where there is a mixed impact. Suppose the purely indirect impact is positive and we want to know whether it can be offset by other components of the mediated probabilistic relation. If \( Y_1 \) screens off \( x \) from \( z \), and each of the other layers of partitions screens off all members of the immediately preceding partition from \( z \), then the issue is settled. The positive indirect impact is not offset, so that \( x \) confirms \( z \) overall. Suppose, however, that the partition \( Y_j \) screens off only the negative impact of each member of \( Y_{j-1} \) from \( z \), allowing some member \( y_{j-1,i_{j+1}} \) of \( Y_{j-1} \) to have a positive \( y_{j,i_j} \)-bypass impact on \( z \). The trouble is that even if the bypass part of the impact is positive, the mixed impact, of which the positive bypass impact is only one factor, can still be negative if the indirect part of the impact that \( x \) has on \( y_{j-1,i_{j+1}} \) through the partial chains of propositions is negative. To make this easier to see, we may rewrite the second line of (7) as follows:

\[
\sum_{i_1=1}^{m_1} \cdots \sum_{i_m=1}^{m_m} \Pr(y_{m_1,i_1} \mid y_{m-1,i_m}) C_D(z; y_{m-1,i_m} \mid y_{m_1,i_1}) C_D(y_{m-2,i_{m-1}}; y_{m-1,i_m}) \cdots C_D(y_{1,i_1}; x)
= \sum_{i_1=1}^{m_1} \cdots \sum_{i_m=1}^{m_m} \Pr(y_{m_1,i_1} \mid y_{m-1,i_m}) C_D(z; y_{m-1,i_m} \mid y_{m_1,i_1}) \sum_{i_{m-1}=1}^{m_{m-1}} \cdots \sum_{i_1=1}^{m_1} C_D(y_{m-2,i_{m-1}}; y_{m-1,i_m}) \cdots C_D(y_{1,i_1}; x)
\]

Let’s suppose \( Y_m \) screens off each member of \( Y_{m-1} \) from \( z \) except that \( C_D(z; y_{m-1,i_m} \mid y_{m_1,i_1}) \) is positive. Although \( y_{m-1,i_m} \) has a positive \( y_{m,i_m} \)-bypass impact on \( z \), the mixed impact \( \Pr(y_{m_1,i_1} \mid y_{m-1,i_m}) C_D(z; y_{m-1,i_m} \mid y_{m_1,i_1}) \sum_{i_{m-1}=1}^{m_{m-1}} \cdots \sum_{i_1=1}^{m_1} C_D(y_{m-2,i_{m-1}}; y_{m-1,i_m}) \cdots C_D(y_{1,i_1}; x) \) can still be negative if \( \sum_{i_{m-1}=1}^{m_{m-1}} \cdots \sum_{i_1=1}^{m_1} C_D(y_{m-2,i_{m-1}}; y_{m-1,i_m}) \cdots C_D(y_{1,i_1}; x) \) is negative, that is, if the total amount of indirect impact \( x \) has on \( y_{m-1,i_m} \) through the \( n_1 \times \cdots \times n_m \) partial chains of propositions from \( x \) up to \( y_{m-1,i_m} \) is negative. So, even if the purely indirect impact \( x \) has on \( z \) is positive, it can be offset by a negative mixed impact. This means that we cannot always replace the screening-off condition by the negative impact screening-off condition for the purpose of ensuring the absence of the offsetting negative impact.

Before concluding the paper, we mention one special case where each of \( m \) layers of partitions has only two members, that is, \( Y_1 = \{y_1, -y_1\}, \ldots, Y_m = \{y_m, -y_m\} \). This means that there are \( 2^m \) chains of propositions that connect \( x \) to \( z \). We can draw a surprisingly strong conclusion from a small amount of information in the case. Let’s suppose \( x \) confirms \( y_1, y_j \) confirms \( y_{j+1} \) for any \( j = 1, \ldots, m-1 \); and \( y_m \) confirms \( z \). Based on this information on one chain of propositions alone, we can tell that \( x \) confirms \( z \) through each of the \( 2^m \) chains of propositions. Note first that any ‘cross impact’ that \( y_j \) has on \( -y_{j+1} \) in the next layer is negative since \( y_j \)
confirms \( y_{j+1} \) by the supposition. It follows from this that any ‘parallel impact’ \( \neg y_{j} \) has on \( \neg y_{j+1} \) in
the next layer is positive. It follows further that any cross impact in the opposite direction that
\( \neg y_{j} \) has on \( y_{j+1} \) in the next layer is negative. Consequently, \( y_{1} \) confirms \( y_{m} \) through any partial
chain between them, and \( \neg y_{1} \) confirms \( \neg y_{m} \) through any partial chain between them. This is
because any partial chain that comes back to the same side has an even number of cross impacts
that are negative, making the resulting indirect impact positive by the anti-transitivity of
disconfirmation. It also follows that \( y_{1} \) disconfirms \( \neg y_{m} \) through any partial chain between them,
and \( \neg y_{1} \) disconfirms \( y_{m} \) through any partial chain between them. But then \( x \) confirms \( z \) through
any chain because \( x \) confirms \( y_{1} \) and \( y_{m} \) confirms \( z \) (and hence \( x \) disconfirms \( \neg y_{1} \) and \( \neg y_{m} \)
disconfirms \( z \)). If the further condition holds that \( Y_{1} \) screens off \( x \) from \( z \), and \( Y_{j} \) screens off \( y_{j-1} \)
and \( \neg y_{j-1} \) from \( z \) for any \( j = 2, \ldots, m \), then \( x \) confirms \( z \) overall.

8. Conclusion

It is a commonplace to warn a novice in probabilistic reasoning that some features taken for
granted in deductive logic do not hold anymore, and transitivity is one of them. The warning is
important since it appears reasonable, intuitively, that if \( x \) confirms \( y \) and \( y \) in turn confirms \( z \),
then \( x \) confirms \( z \), even if confirmation only means raising the probability. There is, however, a
sense in which this intuition is actually correct even in probabilistic reasoning, namely, if \( x \)
confirms \( y \) and \( y \) in turn confirms \( z \), then \( x \) indirectly confirms \( z \) through \( y \). The indirect
confirmation, as distinguished from the total mediated confirmation, works much like logical
entailment. We can even put together a chain of confirmations; \( x \) confirms \( y_{1} \); \( y_{1} \) in turn confirms
\( y_{2} \); \ldots; and \( y_{m} \) in turn confirms \( z \). It follows without further assumptions that \( x \) indirectly confirms
\( z \) through the chain \( y_{1}, \ldots, y_{m} \).

Of course, even if \( x \) indirectly confirms \( z \) through one chain of propositions, it does not
follow generally that \( x \) confirms \( z \) over all. There are other ways \( x \) can affect the probability of \( z \).
One is the impact \( x \) has on \( z \) through other chains of propositions. So, we need to examine the
indirect impact through each chain of propositions. Fortunately, we can determine for any chain
of propositions whether \( x \) confirms, disconfirms, or neither confirms nor disconfirms \( z \) through
the chain, provided the probabilistic relation in each link of the chain is known. Of interest here
is that in order for the indirect impact through a chain to be positive, it is not necessary for each
link of the chain to be positive. Because of the anti-transitivity of disconfirmation, \( x \) confirms \( z \)
through the chain if the number of the negative links is even (and the impact is not killed in any
link in which the impact is zero).

Other than affecting the probability of \( z \) indirectly through the chains of propositions, \( x \)
can also have a purely bypass impact and mixed impacts on \( z \). Our formula (7) in Section 7
identifies the components and structure of those impacts. We can see from the formula that when
the indirect impact is positive, we can verify that the overall impact is positive by a series of
screening-off conditions. In some cases we can weaken the requirement of the screening-off
condition to the negative impact screening-off condition, but not always. We should not lose
sight of the main principle, though. The indirect confirmation through a chain of propositions is
transitive. We go over other components of the mediated probabilistic relation to see whether the
positive indirect confirmation is offset by them.
Appendix

Appendix A: Proof of (3)

\[ \text{Pr}(z \mid x) - \text{Pr}(z) = [\text{Pr}(z \land y \mid x) + \text{Pr}(z \land \lnot y \mid x)] - [\text{Pr}(z \land y) + \text{Pr}(z \land \lnot y)] \]

\[ = [\text{Pr}(z \land y \mid x) - \text{Pr}(z \land y)] + [\text{Pr}(z \land \lnot y \mid x) - \text{Pr}(z \land \lnot y)] \]

\[ = [\text{Pr}(z \land y)\text{Pr}(y \mid x) - \text{Pr}(z \mid y)\text{Pr}(y)] + [\text{Pr}(z \land \lnot y)\text{Pr}(\lnot y \mid x) - \text{Pr}(z \mid \lnot y)\text{Pr}(\lnot y)] \]

\[ = [[\text{Pr}(z \land y)\text{Pr}(y \mid x) - \text{Pr}(z \mid y)\text{Pr}(y)] + \text{Pr}(z \land y)\text{Pr}(y \mid x) - \text{Pr}(z \mid y)\text{Pr}(y)] \]

\[ + [\text{Pr}(z \land \lnot y)\text{Pr}(\lnot y \mid x) - \text{Pr}(z \mid \lnot y)\text{Pr}(\lnot y)] + [\text{Pr}(z \land \lnot y)\text{Pr}(\lnot y \mid x) - \text{Pr}(z \mid \lnot y)\text{Pr}(\lnot y)] \]

\[ = \text{Pr}(z \mid y)\text{Pr}(y \mid x) - \text{Pr}(y) + \text{Pr}(z \mid \lnot y)\text{Pr}(\lnot y \mid x) - \text{Pr}(z \mid \lnot y) \]

\[ + \text{Pr}(y \mid x)[\text{Pr}(z \mid y \land x) - \text{Pr}(z \mid y)] + \text{Pr}(\lnot y \mid x)[\text{Pr}(z \mid \lnot y \land x) - \text{Pr}(z \mid \lnot y)] \]

\[ = \text{Pr}(z \mid y)\text{Pr}(y \mid x) - \text{Pr}(y) + \text{Pr}(z \mid \lnot y)\text{Pr}(\lnot y \mid x) - \text{Pr}(z \mid \lnot y) \]

\[ + \text{Pr}(y \mid x)[\text{Pr}(z \mid y \land x) - \text{Pr}(z \mid y)] + \text{Pr}(\lnot y \mid x)[\text{Pr}(z \mid \lnot y \land x) - \text{Pr}(z \mid \lnot y)] \]

\[ = \text{Pr}(z \mid y)\text{Pr}(y \mid x) - \text{Pr}(y) + \text{Pr}(z \mid \lnot y)\text{Pr}(\lnot y \mid x) - \text{Pr}(z \mid \lnot y) \]

\[ + \text{Pr}(y \mid x)[\text{Pr}(z \mid y \land x) - \text{Pr}(z \mid y)] + \text{Pr}(\lnot y \mid x)[\text{Pr}(z \mid \lnot y \land x) - \text{Pr}(z \mid \lnot y)] \]

\[ + \text{Pr}(z \land y)\text{Pr}(y \mid x) - \text{Pr}(z \mid y)\text{Pr}(y)] + [\text{Pr}(z \land \lnot y)\text{Pr}(\lnot y \mid x) - \text{Pr}(z \mid \lnot y)\text{Pr}(\lnot y)] \]

\[ = \text{Pr}(z \mid y)\text{Pr}(y \mid x) - \text{Pr}(y) + \text{Pr}(z \mid \lnot y)\text{Pr}(\lnot y \mid x) - \text{Pr}(z \mid \lnot y) \]

\[ + \text{Pr}(y \mid x)[\text{Pr}(z \mid y \land x) - \text{Pr}(z \mid y)] + \text{Pr}(\lnot y \mid x)[\text{Pr}(z \mid \lnot y \land x) - \text{Pr}(z \mid \lnot y)] \]

Applying (3), we get:\n
\[ C_D(z; x) = \text{Pr}(z \mid x) - \text{Pr}(z) \]

\[ = \sum_{i=1}^{n} \text{Pr}(z \land y_i \mid x) - \sum_{i=1}^{n} \text{Pr}(z \land y_i) \]

\[ = \sum_{i=1}^{n} [\text{Pr}(z \land y_i \mid x) - \text{Pr}(z \land y_i)] \]

\[ = \sum_{i=1}^{n} [\text{Pr}(z \mid y_i \land x)\text{Pr}(y_i \mid x) - \text{Pr}(z \mid y_i)\text{Pr}(y_i)] \]

\[ = \sum_{i=1}^{n} [\text{Pr}(z \mid y_i \land x)\text{Pr}(y_i \mid x) - \text{Pr}(z \mid y_i)\text{Pr}(y_i)] \]

\[ + \text{Pr}(z \mid y_i)\text{Pr}(y_i \mid x) - \text{Pr}(z \mid y_i) \text{Pr}(y_i)] + [\text{Pr}(z \mid \lnot y_i)\text{Pr}(\lnot y_i \mid x) - \text{Pr}(z \mid \lnot y_i)\text{Pr}(\lnot y_i)] \]

\[ = \sum_{i=1}^{n} \text{Pr}(y_i \mid x)\text{Pr}(z \mid y_i \land x) - \text{Pr}(z \mid y_i)\text{Pr}(y_i)] \]

\[ + \sum_{i=1}^{n} \text{Pr}(y_i \mid x)\text{Pr}(z \mid y_i \land x) - \text{Pr}(z \mid y_i)\text{Pr}(y_i)] \]

\[ + \sum_{i=1}^{n} \text{Pr}(y_i \mid x)\text{Pr}(z \mid y_i \land x) - \text{Pr}(z \mid y_i)\text{Pr}(y_i)] \]

\[ + \sum_{i=1}^{n} \text{Pr}(y_i \mid x)\text{Pr}(z \mid y_i \land x) - \text{Pr}(z \mid y_i)\text{Pr}(y_i)] \]

\[ = \sum_{i=1}^{n} C_D(z; y_i)C_D(y_i; x) + \sum_{i=1}^{n} \text{Pr}(y_i \mid x)C_D(z; x \mid y_i) \]
Appendix C: Proof of (6)

The proof has \( m \) steps and each step uses (5) above. In the first step, we take \( z \) to be the disjunction of the \( n_m \times \cdots \times n_2 \) pairwise incompatible conjunctions \( z \land y_{m,i} \land \cdots \land y_{2,i} \). By (5) the total impact \( x \) has on each conjunction \( z \land y_{m,i} \land \cdots \land y_{2,i} \) is:

\[
\Pr(z \land y_{m,i} \land \cdots \land y_{2,i} \mid x) - \Pr(z \land y_{m,i} \land \cdots \land y_{2,i})
= \sum_{i=1}^{n_1} \Pr(z \land y_{m,i} \land \cdots \land y_{2,i} \land y_{1,i} \mid x) - \sum_{i=1}^{n_1} \Pr(z \land y_{m,i} \land \cdots \land y_{2,i} \land y_{1,i})
= \sum_{i=1}^{n_1} C_D(z \land y_{m,i} \land \cdots \land y_{2,i} ; y_{1,i}) C_D(y_{1,i} ; x)
+ \sum_{i=1}^{n_1} \Pr(y_{1,i} \mid x) C_D(z \land y_{m,i} \land \cdots \land y_{2,i} ; y_{1,i} \mid x)
\]

(8)

We plug in (8) to \( \Pr(z \mid x) - \Pr(z) \) as follows:

\[
\Pr(z \mid x) - \Pr(z)
= \sum_{i=1}^{n_1} \sum_{i=1}^{n_1} \Pr(z \land y_{m,i} \land \cdots \land y_{1,i} \mid x) - \sum_{i=1}^{n_1} \sum_{i=1}^{n_1} \Pr(z \land y_{m,i} \land \cdots \land y_{1,i})
= \sum_{i=1}^{n_1} \sum_{i=1}^{n_1} [\sum_{i=1}^{n_1} \Pr(z \land y_{m,i} \land \cdots \land y_{1,i} \mid x) - \sum_{i=1}^{n_1} \Pr(z \land y_{m,i} \land \cdots \land y_{1,i})]
= \sum_{i=1}^{n_1} \sum_{i=1}^{n_1} [\sum_{i=1}^{n_1} C_D(z \land y_{m,i} \land \cdots \land y_{2,i} ; y_{1,i}) C_D(y_{1,i} ; x)
+ \sum_{i=1}^{n_1} \Pr(y_{1,i} \mid x) C_D(z \land y_{m,i} \land \cdots \land y_{2,i} ; y_{1,i} \mid x)]
\]

(9)

In the second step we take \( z \) to be the disjunction of the \( n_m \times \cdots \times n_3 \) pairwise incompatible conjunctions \( z \land y_{m,i} \land \cdots \land y_{3,i} \). By (5) the total impact \( y_{1,i} \) has on each conjunction \( z \land y_{m,i} \land \cdots \land y_{3,i} \) is:

\[
\Pr(z \land y_{m,i} \land \cdots \land y_{3,i} \mid y_{1,i}) - \Pr(z \land y_{m,i} \land \cdots \land y_{3,i})
= \sum_{i=1}^{n_1} \Pr(z \land y_{m,i} \land \cdots \land y_{3,i} \land y_{2,i} \mid y_{1,i}) - \sum_{i=1}^{n_1} \Pr(z \land y_{m,i} \land \cdots \land y_{3,i} \land y_{2,i})
= \sum_{i=1}^{n_1} C_D(z \land y_{m,i} \land \cdots \land y_{3,i} ; y_{2,i}) C_D(y_{2,i} ; y_{1,i})
+ \sum_{i=1}^{n_1} \Pr(y_{2,i} \mid y_{1,i}) C_D(z \land y_{m,i} \land \cdots \land y_{3,i} ; y_{2,i} \mid y_{1,i})
\]

(10)

We plug in (10) to (9) as follows:
\[
\Pr(z \mid x) - \Pr(z) = \sum_{i=1}^{n_i} \cdots \sum_{i=n} \Pr(y_{i,j} \mid y_{j-1,j-1}, \cdots, y_{j-2,j-2}) C_D(z; y_{j-1,j-1}, \cdots, y_{j-2,j-2}) C_D(y_{j-1}; x) \\
+ \sum_{i=1}^{n_i} \cdots \sum_{i=n} \Pr(y_{i,j} \mid y_{j-1,j-1}, \cdots, y_{j-2,j-2}) C_D(z; y_{j-1,j-1}, \cdots, y_{j-2,j-2}) C_D(y_{j-1}; x)
\]

We apply the same process to each of the \(m\) layers of partitions. When we apply it to the \(j\)-th layer of partition, the new multiplicand \(C_D(y_{j,i}, y_{j-1,i-1})\) appears in the first line and the following line is added.

\[
\sum_{i=1}^{n_i} \cdots \sum_{i=n} \Pr(y_{j,i} \mid y_{j-1,i-1}) C_D(z; y_{j-1,i-1}) \prod_{k=1}^{j-2} C_D(y_{k+1,i-1}; y_{k,i}) C_D(y_{j-1}; x)
\]

The final result is (6).

\[
\Pr(z \mid x) - \Pr(z) = \sum_{i=1}^{n_i} \cdots \sum_{i=n} C_D(z; y_{m,i}) \cdots C_D(y_{m,i}; y_{m-1,i-1}) \cdots C_D(y_{j,i}; x) \\
+ \sum_{j=1}^{m} \sum_{i=1}^{n_i} \cdots \sum_{i=n} \Pr(y_{j,i} \mid y_{j-1,i-1}) C_D(z; y_{j-1,i-1}) \prod_{k=1}^{j-1} C_D(y_{k+1,i-1}; y_{k,i}) C_D(y_{j-1}; x) \\
+ \sum_{i=1}^{n_i} \cdots \sum_{i=n} \Pr(y_{i,i} \mid x) C_D(z; x) C_D(y_{i,i}; x)
\]

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